LANDAUŮV DIAMAGNETISMUS

počet $\bar{e}$ v krystalu: $N = \sum_s \sum_{\vec{k}} n_{s\vec{k}} = 2 \frac{\mathcal{V}}{(2\pi)^3} \int d\vec{k} \, \frac{1}{e^{\beta(\varepsilon_k - \mu)} + 1}$ $f_0(\varepsilon)$ ... F–D

$2 \frac{1}{V_k} d\vec{k}$ ... počet stavů v $d\vec{k}$ / . distrib. fce $f_0(\varepsilon)$ viz FPPO (str. 4–5)

koncentrace $\bar{e}$: $n = \frac{N}{\mathcal{V}} = 2 \int \frac{d\vec{k}}{(2\pi)^3} f_0(\varepsilon) = \frac{1}{\pi^2} \int_0^\infty dk \, k^2 f_0(\varepsilon)$

chemický potenciál μ závisí na T a n

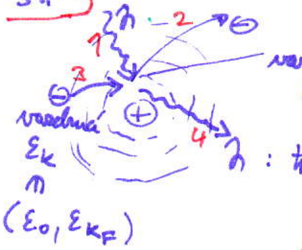
při $T=0$: $f_0(\varepsilon_k) = \Theta(\mu - \varepsilon_k)$, hraniční hodnota je Fermiho energie $E_F = \frac{\hbar^2 k_F^2}{2m} = \mu_0$

$$\boxed{n = \frac{1}{\pi^2} \int_0^{k_F} k^2 dk = \frac{k_F^3}{3\pi^2}}$$ .... n v kovech $\sim 10^{28} - 10^{29}\ m^{-3}$

$E_F \sim 1 - 15\,eV$

$kT|_{300K} \sim 10\,meV$ $\Rightarrow E_F \gg kT$

měření:



γ : $\hbar\omega = \varepsilon_k - E_b$

$\varepsilon_k \in (\varepsilon_0, \varepsilon_{k_F})$

$\Downarrow$

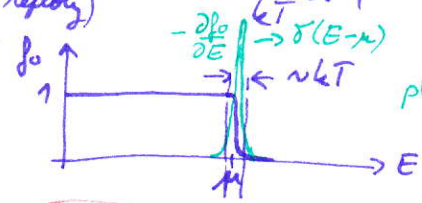
z šířky spektra získám $E_F = \varepsilon_{k_F} - \varepsilon_0$

hustota stavů

$$\boxed{n = 2 \int \frac{d\vec{k}}{(2\pi)^3} \int_0^\infty dE \, \delta(E - \varepsilon_k) f_0(E) = \int_0^\infty dE \, g(E) f_0(E)}$$

$g(E) = 2 \int \frac{d\vec{k}}{(2\pi)^3} \delta(E - \varepsilon_k)$ ... hustota stavů

vezmeme (nízké teploty) kovy: $\beta\mu = \frac{\mu}{kT} \gg 1$



pro volné elektrony: $g(E) = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \sqrt{E}$

platí: $\int_0^\infty \left(-\frac{\partial f_0}{\partial E}\right) dE = -\int_0^\infty df_0 = f_0(0) - f_0(\infty) = 1$

platí: $\boxed{I = \int_0^\infty \chi(E) f_0(E) \, dE} = \int_0^\infty f_0(E) \, d\varphi(E) \overset{P.P.}{=} \left[f_0(E) \varphi(E) \right]_0^\infty - \int_0^\infty \varphi(E) \frac{\partial f_0}{\partial E} dE =$

$\chi(E) = \frac{d\varphi(E)}{dE}$

$= -\varphi(0) + \boxed{\int_0^\infty \varphi(E) \left(-\frac{\partial f_0}{\partial E}\right) dE = I}$

$\varphi(0) = 0$ v rozumných případech

zvolíme-li $\chi(E) = g(E) \Rightarrow I = n$

$\varphi(E) = \int g(E) dE = \frac{1}{3\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2} + \varphi(0)$, $\varphi(0) = 0$

při $T=0$: $-\frac{\partial f_0}{\partial E} = \delta(E - \mu_0) \Rightarrow I = \int_0^\infty \varphi(E) \, \delta(E - \mu_0) dE = \frac{1}{3\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \mu_0^{3/2} = n$

$$\boxed{\mu_0 = \left[3\pi^2 n \left(\frac{\hbar^2}{2m}\right)^{3/2} \right]^{2/3} = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}}$$

při $T \neq 0$ a $\beta\mu \gg 1$:

jelikož $-\frac{\partial f_0}{\partial E} \approx \delta(E - \mu)$, tak v $\int$ jsou podstatné jen příspěvky od $\varphi(E)$ v okolí μ, zavedu proměnnou $x = \beta(E - \mu)$ a rozvinu $\varphi(E) = \psi(x)$ kolem $x = 0$

$\psi(x) = \psi(0) + \psi'(0) x + \frac{\psi''(0)}{2} x^2 + \dots$

$-\frac{\partial f_0}{\partial E} dE = -\frac{\partial f_0}{\partial x} dx = -\frac{\partial}{\partial x} \left(\frac{1}{1 + e^x} \right) dx = \frac{e^x}{(1 + e^x)^2} dx \cdot \frac{e^{-2x}}{e^{-2x}} = \frac{e^{-x}}{(1 + e^{-x})^2} dx$ sudá!

$$I = \int_{-\beta\mu}^{\infty} \psi(x)\left(-\frac{\partial f_0}{\partial x}\right)dx \overset{\text{rozvoj}}{=} \psi(0)\underbrace{\int_{-\infty}^{\infty}\left(-\frac{\partial f_0}{\partial x}\right)dx} + \psi'(0)\boxed{\int_{-\infty}^{\infty}\underbrace{\left(-\frac{\partial f_0}{\partial x}\right)}_{\text{sudá}}\overbrace{x}^{\text{lichá}}\,dx}_{=0} + \frac{\psi''(0)}{2}\int_{-\infty}^{\infty}\frac{x^2 e^{-x}}{(1+e^{-x})^2}dx =$$

$-\beta\mu \to -\infty$ (už se moc nezplete, příspěvky k $\int$ jsou pro $x < -\beta\mu$ zanedbatelné)

$$\int_{-\beta\mu}^{\infty}\left(-\frac{\partial f_0}{\partial x}\right)dx = \int_0^{\infty}\left(-\frac{\partial f_0}{\partial E}\right)dE = 1$$

$$= \psi(0) + \psi''(0)\underbrace{\int_0^{\infty}\frac{x^2 e^{-x}}{(1+e^{-x})^2}dx} =$$

$$= \int_0^{\infty} x^2 e^{-x}\left[1 - 2e^{-x} + 3e^{-2x} \ldots\right]dx = -\int_0^{\infty} x^2 \sum_{n=1}^{\infty}(-1)^n n\, e^{-nx}dx$$

$$\left(\frac{1}{(1+y)^2} \approx 1 - 2y + 3y^2 \ldots \qquad ' = -2\frac{1}{(1+y)^3} \;/\; ' \; 6\frac{1}{(1+y)^4}\right)$$

$$= \psi(0) - \psi''(0)\sum_{n=1}^{\infty}(-1)^n n \underbrace{\int_0^{\infty} x^2 e^{-nx}dx} =$$

$$= \underbrace{\left[x^2\frac{e^{-nx}}{-n}\right]_0^{\infty}}_{0} - \int_0^{\infty} 2x\frac{e^{-nx}}{-n}dx = \frac{2}{n}\int_0^{\infty} x e^{-nx}dx = \frac{2}{n}\underbrace{\left[x\frac{e^{-nx}}{-n}\right]_0^{\infty}}_{0}$$

$$-\frac{2}{n}\int_0^{\infty}\frac{e^{-nx}}{-n}dx = \frac{2}{n^2}\int_0^{\infty} e^{-nx}dx = -\frac{2}{n^3}\left[e^{-nx}\right]_0^{\infty} = \frac{2}{n^3}$$

$$= \psi(0) - \psi''(0)\sum_{n=1}^{\infty}(-1)^n\frac{2}{n^2} = \boxed{\psi(0) + \frac{\pi^2}{6}\psi''(0) = I}$$

$$\psi(x) = \varphi(E)\Big|_{E=\frac{x}{\beta}+\mu} = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\frac{x}{\beta}+\mu\right)^{3/2} \Rightarrow n = I$$

$$n = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\mu^{3/2} + \frac{\pi^2}{6}\frac{3}{4\beta^2}\mu^{-1/2}\right] = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu^{3/2}\left[1 + \frac{\pi^2}{8\beta^2}\frac{1}{\mu^2}\right]$$

$$\mu = \left[n\, 3\pi^2\left(\frac{\hbar^2}{2m}\right)^{3/2}\right]^{2/3}\underbrace{\left(1 + \frac{\pi^2}{8\beta^2}\frac{1}{\mu_0^2}\right)^{-2/3}} =$$

$\frac{1}{\beta^2} \ll 1$ … nahradím $\mu = \mu_0$ a moc to nezměním

$\approx 0 \Rightarrow$ rozvoj: $\left(1 + \frac{\pi^2}{8}y\right)^{-2/3} \approx 1 - \frac{\pi^2}{12}y$

$$= \underbrace{(3\pi^2 n)^{2/3}\frac{\hbar^2}{2m}}_{\mu_0}\left[1 - \frac{\pi^2}{12}\frac{1}{\beta^2\mu_0^2}\right] =$$

$$' = -\frac{2}{3}(\ )^{-5/3}\cdot\frac{\pi^2}{8}\Big|_0 = -\frac{1}{12}\pi^2$$

$$= \boxed{\mu_0\left(1 - \frac{\pi^2}{12}\frac{1}{\beta^2\mu_0^2}\right) = \mu} \overset{\mu_0 = E_F}{=} E_F - \frac{\pi^2}{12}\frac{k_B^2 T^2}{E_F}$$

… při nízkých teplotách klesá chemický potenciál kvadraticky s teplotou

měrné teplo

(kinetická) energie e^- plynu: $U = \sum_s \sum_{\vec{k}} \varepsilon_{s\vec{k}}\, n_{s\vec{k}} = \int_0^{\infty} E\, g(E)\, f_0(E)\, dE$

při $T=0$: $U = \int_0^{\infty} E\, g(E)\,\theta(\mu_0 - E)\,dE = \int_0^{\mu_0} E\, g(E)\,dE \underset{\mu_0 = E_F}{=} \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^{E_F} E^{3/2}\,dE = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F E_F^{3/2}\cdot\frac{2}{5}$

$$n\big|_{T=0} = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{3/2}$$

$$= \boxed{\frac{3}{5} n E_F = U\big|_{T=0}}$$

při $T \neq 0$, $\mu\beta \gg 1$: stejný trik jako pro μ: $I = \int_0^{\infty} \chi(E) f_0(E)\,dE = U$

$\chi(E) = E\, g(E)$

$$\Psi(E) = \int X(E)\,dE = \int E\,g(E)\,dE = \int \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}\,dE = \frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{5/2}$$

$$\Psi(x) = \frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left(\frac{x}{\beta}+\mu\right)^{5/2} \quad \Big/'' = \frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{5}{2}\frac{3}{2}\frac{1}{\beta^2}\left(x/\beta+\mu\right)^{1/2}$$

$$U = \Psi(0) + \frac{\pi^2}{6}\Psi''(0) = \frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\mu^{5/2} + \frac{\pi^2}{6}\frac{15}{4}\frac{1}{\beta^2}\mu^{1/2}\right] = \frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu^{5/2}\left[1 + \frac{5\pi^2}{8}\frac{1}{\beta^2}\frac{1}{\mu^2}\right]$$

$$\mu^{5/2} = \mu_0^{5/2}\left[1 - \frac{\pi^2}{12}\frac{1}{\beta^2\mu_0^2}\right]^{5/2} \approx \mu_0^{5/2}\left[1 - \frac{5}{24}\pi^2\frac{1}{\beta^2\mu_0^2}\right]$$

$\approx 0 \Rightarrow$ TAYLOR $\left(1-\frac{\pi^2}{12}y\right)^{5/2} \approx 1 - \frac{5}{24}\pi^2 y$ $\qquad {}' = \frac{5}{2}(\;)^{3/2}\left(-\frac{\pi^2}{12}\right)$

$\frac{1}{\beta^2} \ll 1 \Rightarrow \mu \doteq \mu_0$

zanedbám člen $\frac{1}{\beta^4\mu_0^4}$

$$U = \frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu_0^{5/2}\underbrace{\left(1-\frac{5}{24}\pi^2\frac{1}{\beta^2\mu_0^2}\right)\left(1+\frac{5\pi^2}{8}\frac{1}{\beta^2\mu_0^2}\right)}_{1+\frac{\pi^2}{\beta^2\mu_0^2}\left(\frac{15}{24}-\frac{5}{24}\right)} \approx \boxed{\underbrace{\frac{1}{5\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu_0^{5/2}}_{U|_{T=0} \neq f(T)}\left(1+\frac{5}{12}\frac{\pi^2}{\beta^2\mu_0^2}\right) = U}$$

$$U - U|_{T=0} = \frac{1}{12}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{1}{\beta^2}\mu_0^{1/2} = \frac{1}{12}\left(\frac{2m}{\hbar^2}\right)^{3/2} k_B^2\mu_0^{1/2}T^2$$

$$\boxed{c_V = \frac{\partial(U-U|_{T=0})}{\partial T}} = \boxed{\underbrace{\frac{1}{6}\left(\frac{2m}{\hbar^2}\right)^{3/2}k_B^2\mu_0^{1/2}}_{\gamma}T} \boxed{= \frac{\pi^2}{3}g_F k_B^2 T}$$

$$\boxed{g_F = g(E_F) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E_F} = \frac{mk_F}{\pi^2\hbar^2}} \quad \sqrt{E_F} = \frac{\hbar k_F}{(2m)^{1/2}}$$

hustota stavů na Fermiho mezi

Magnetické vlastnosti volných e^-

SPINOVÁ SUSCEPTIBILITA

v mg. poli je energie e^- modifikovaná díky spinu

$$\varepsilon_{\uparrow\vec{k}} = \frac{\hbar^2k^2}{2m} - \mu_B B \qquad \varepsilon_{\downarrow\vec{k}} = \frac{\hbar^2k^2}{2m} + \mu_B B$$

$\mu_B = \frac{e\hbar}{2m}$ ... Bohrův magneton

viz Lukáš: $V_m = -\mu_B g_J M_J B$

$$g_J = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)} = 2 \quad S=\tfrac{1}{2},\ L=0,\ J=\tfrac{1}{2}$$

$$V_m = -\mu_B \cdot 2 \cdot \begin{matrix}\uparrow 1/2\\ \downarrow -1/2\end{matrix}\; B$$

koncentrace e^- v obou spinových podsystémech je různá:
(při $T=0$)

$$n_\uparrow = \frac{n}{2}(1+p) \equiv \frac{k_\uparrow^3}{6\pi^2}, \qquad n_\downarrow = \frac{n}{2}(1-p) \equiv \frac{k_\downarrow^3}{6\pi^2}$$

p je bezrozměrná míra polarizace: $n_\uparrow + n_\downarrow = n$, $n_\uparrow - n_\downarrow = np$

dosazením $n|_{T=0} = \frac{k_F^3}{3\pi^2}$ dostaneme:

$$n_\uparrow = \frac{k_F^3}{6\pi^2}(1+p) = \frac{k_\uparrow^3}{6\pi^2} \Rightarrow k_\uparrow = k_F(1+p)^{1/3}$$

$$n_\downarrow = \frac{k_F^3}{6\pi^2}(1-p) = \frac{k_\downarrow^3}{6\pi^2} \Rightarrow k_\downarrow = k_F(1-p)^{1/3}$$

chemický potenciál obou podsystémů je stejný:

$$\frac{\hbar^2k_\uparrow^2}{2m} - \mu_B B = \mu = \frac{\hbar^2k_\downarrow^2}{2m} + \mu_B B$$

$$\frac{\hbar^2}{2m}\left(k_\uparrow^2 - k_\downarrow^2\right) = 2\mu_B B \Rightarrow E_F\underbrace{\left((1+p)^{2/3} - (1-p)^{2/3}\right)}_{\approx 1+\frac{2}{3}p-\left(1-\frac{2}{3}p\right) = \frac{4}{3}p} = 2\mu_B B \Rightarrow \boxed{p \approx \frac{3\mu_B B}{2E_F}}$$

hustota energie mg. pole v látce: $U = \frac{1}{2}HB = \frac{B^2}{2\mu_0(1+\chi)} \approx \frac{B^2}{2\mu_0} - \chi\frac{B^2}{2\mu_0}$

energie mg. pole ve vakuu — energie mané interakce s látkou

hustota energie systému e^- v hodu za přítomnosti mg. pole:

$$U_B\Big|_{T=0} = \frac{3}{5} n_\uparrow E_{F\uparrow} - n_\uparrow \mu_B B + \frac{3}{5} n_\downarrow E_{F\downarrow} + n_\downarrow \mu_B B = \frac{3}{5}\frac{n}{2}\frac{\hbar^2 k_F^2}{2m}\left((1+p)^{5/3}+(1-p)^{5/3}\right)$$

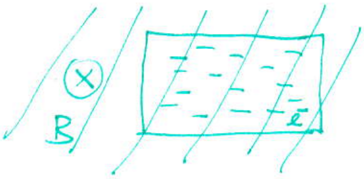
$E_{F\uparrow} = \frac{\hbar^2 k_\uparrow^2}{2m}$, $E_{F\downarrow} = \frac{\hbar^2 k_\downarrow^2}{2m}$

$p \approx 0$: $1+\frac{5}{3}p+\frac{1}{2}\frac{5}{3}\frac{2}{3}p^2 \dots + 1 - \frac{5}{3}p + \frac{5}{9}p^2$

pozn.: pokud bychom nedosadili p ale provedli $\frac{\partial U_B}{\partial p}=0$ došli bychom ke stejnému výsledku $p = \frac{3\mu_B B}{2E_F}$ tzn. v rovnováze se ustaví takové rozdělení e^- mezi obě polarizace spinu aby U_B bylo minimální!

$$- np\mu_B B \approx \frac{3}{5} n E_F \frac{1}{2}\left(2+\frac{10}{9}p^2\right) - np\mu_B B = \underbrace{\frac{3}{5} n E_F}_{U|_{B=0,\,T=0} = U_0} + \frac{1}{3} n E_F p^2 - np\mu_B B =$$

$$= U_0 + \frac{3}{4}\frac{\mu_B^2 B^2}{E_F^2} n E_F - n\frac{3\mu_B^2 B^2}{2E_F} = \boxed{U_0 - \frac{B^2}{2\mu_0}\frac{3}{2}\frac{n\mu_0\mu_B^2}{E_F} = U_B}$$



celková energie = E mg. pole ve vakuu + E e^- bez mg. pole + (E e^- v mg. poli − E e^- bez mg. pole)

$U_B - U_0$ = INTERAKCE $= -\chi\frac{B^2}{2\mu_0}$

$$U_B - U_0 = -\chi\frac{B^2}{2\mu_0} = -\frac{B^2}{2\mu_0}\frac{3}{2}\frac{n\mu_0\mu_B^2}{E_F}$$

$$\chi = \frac{3}{2}\frac{n\mu_0\mu_B^2}{E_F} = \frac{\mu_0\mu_B^2 k_F m}{\hbar^2\pi^2} = \boxed{\mu_0\mu_B^2 g_F = \chi_{PAULI}}$$

$n = \frac{k_F^3}{3\pi^2}$, $E_F = \frac{\hbar^2 k_F^2}{2m}$

PAULIHO PARAMAGNETISMUS

ORBITÁLNÍ SUSCEPTIBILITA

interakce mg. pole s nábojem e^- ovlivňuje pohyb e^- => změna energie

mějme $\vec{B} = (0,0,B)$

$\vec{A} = (0, B\cdot x, 0)$ ... Landauova kalibrace

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 = \frac{1}{2m}\left(p_x^2 + (p_y + m\omega_c x)^2 + p_z^2\right) \quad ; \quad \omega_c = -\frac{eB}{m} \text{ cyklotronová frekvence}$$

$(e<0)$

co nám řekne klasická mechanika?

HKR: $\dot{\vec{R}} = \frac{\partial H}{\partial \vec{p}}$ $\quad \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{R}}$

$\dot{x} = \frac{p_x}{m}$ $\quad \dot{p}_x = -\omega_c(p_y + m\omega_c x)$

$\dot{y} = \frac{p_y}{m} + \omega_c x$ $\quad \dot{p}_y = 0$

$\dot{z} = \frac{p_z}{m}$ $\quad \dot{p}_z = 0$

=> p_y, p_z jsou integrály pohybu

ve směru y, z se e^- „chová jako volná částice"

integrací rovnic obdržíme: $z = z_0 + \frac{p_z}{m}t$ nezávisle na mg. poli

$x = -\frac{p_y}{m\omega_c} + R\cos\omega_c t$

$y = y_0 + R\sin\omega_c t$

pohyb po kružnici o poloměru R

a teď kvantovka: hledáme vlnovou funkci e^- v mg. poli $\psi(\vec{r}) = e^{ik_y y} e^{ik_z z}\psi(x)$

SR: $H\psi(\vec{r}) = E\psi(\vec{r})$

$$\frac{1}{2m}\left[-\hbar^2\frac{\partial^2}{\partial x^2}+\left(-i\hbar\frac{\partial}{\partial y}+m\omega_c x\right)^2-\hbar^2\frac{\partial^2}{\partial z^2}\right]e^{ik_y y}e^{ik_z z}\psi(x) = E\,e^{ik_y y}e^{ik_z z}\psi(x)$$

$$\frac{1}{2m}\left[-\hbar^2\frac{\partial^2}{\partial x^2}+\left(-i\hbar(ik_y)+m\omega_c x\right)^2+\hbar^2k_z^2\right]\psi(x) = E\psi(x)$$

$$\frac{\hbar^2}{2m}\left[-\frac{\partial^2}{\partial x^2}+\underbrace{\left(k_y+\frac{m\omega_c}{\hbar}x\right)^2}\right]\psi(x) = \left(E-\frac{\hbar^2k_z^2}{2m}\right)\psi(x)$$

potenciální energie má tvar: $\left(\frac{m\omega_c}{\hbar}\right)^2\left(\frac{\hbar k_y}{m\omega_c}+x\right)^2\cdot\frac{\hbar^2}{2m} = \frac{1}{2}k_c\left(x+\frac{\hbar k_y}{m\omega_c}\right)$

$\omega_c^2 = \frac{k_c}{m}$ tuhost

harmonický oscilátor sechcí v $\bar{x} = -\frac{\hbar k_y}{m\omega_c}$

$$E-\frac{\hbar^2k_z^2}{2m} = \hbar\omega_c\left(j+\frac{1}{2}\right), \quad \boxed{j = 0,1,2,\dots}$$

$$\boxed{E_{k_z j} = \frac{\hbar^2k_z^2}{2m}+\hbar\omega_c\left(j+\frac{1}{2}\right)}$$

mějme elektrony v kvádru o objemu $\Omega = L_x L_y L_z$

hustota stavů: $g(E) = \frac{2}{\Omega}\sum_{k_y}\sum_{k_z}\sum_{j=0}^{\infty}\delta(E-E_{k_z j})$, energie nezávisí na k_y => nutno sečíst $\sum_{k_y}$

aby stav ležel v kvádru: $-\frac{1}{2}L_x < \bar{x} < \frac{1}{2}L_x$

$$-\frac{1}{2}L_x < \frac{\hbar k_y}{m\omega_c} < \frac{1}{2}L_x \iff -\frac{m\omega_c L_x}{2\hbar} < k_y < \frac{m\omega_c L_x}{2\hbar}$$

vzdálenost 2 sousedních ~~vlny~~ vlnových čísel je $\frac{2\pi}{L_y}$ =>

=> počet sčítanců v $\sum_{k_y}$ je $\frac{m\omega_c L_x}{\hbar}\cdot\frac{L_y}{2\pi}$

$$g(E) = \frac{m\omega_c L_x L_y}{\pi\hbar\Omega}\sum_{k_z}\sum_{j=0}^{\infty}\delta(E-E_{k_z j}) = \frac{m\omega_c}{2\pi^2\hbar}\sum_{j=0}^{\infty}\int dk_z\,\delta(E-E_{k_z j})$$

$$\sum_{k_z} = \frac{L_z}{2\pi}\int dk_z$$

$$k_z = \frac{\sqrt{2m\left(E_{k_z j}-\hbar\omega_c\left(j+\frac{1}{2}\right)\right)}}{\hbar}$$

$$dk_z = \frac{m\,dE_{k_z j}}{\hbar\sqrt{2m\left(E_{k_z j}-\hbar\omega_c\left(j+\frac{1}{2}\right)\right)}}$$

MÁ TU BÝT $\frac{1}{4\pi^2}$ ALE CHYBU NEVIDÍM

aby byla $\sqrt{\ }$ reálná

$$\boxed{g(E) = \frac{m\omega_c}{2\pi^2\hbar}}\sum_{j=0}^{\infty}\int\frac{m^{1/2}}{\hbar\,2^{1/2}}dE_{k_z j}\frac{1}{\sqrt{E_{k_z j}-\hbar\omega_c\left(j+\frac{1}{2}\right)}}\delta(E-E_{k_z j}) = \boxed{\frac{1}{8\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\hbar\omega_c\sum_{j=0}^{j_{max}}\frac{1}{\sqrt{E-\hbar\omega_c\left(j+\frac{1}{2}\right)}}}$$

hustota energie: $U = \int_0^{\mu}E\,g(E)\,dE \overset{P.P.}{=} \Big[E\underbrace{\int^{E}g(E')dE'}_{g^{(1)}(E)}\Big]_0^{\mu}-\int_0^{\mu}\int^{E}g(E')dE' = \mu g^{(1)}(\mu)-\Big[\underbrace{\int^{E}g^{(1)}(E')dE'}_{g^{(2)}(E)}\Big]_0^{\mu}$

$= \mu g^{(1)}(\mu)-g^{(2)}(\mu)$, $g^{(1)}(\mu)$ pro fermiho energii = N nezávisle na mg. poli, tedy $g^{(1)}(\mu) = g_0^{(1)}(\mu_0)$

$U = U_0+\delta U$ (energie bez mg. pole; odchylka vlivem mg. pole)

$$\delta U = \delta\left[\mu g^{(1)}(\mu)\right]-\delta\left[g^{(2)}(\mu)\right] =$$

$$\to g^{(2)}(\mu_0)+\frac{\partial g^{(2)}}{\partial E}\Big|_{\mu_0}(\mu-\mu_0)\ /\ \delta\ = \delta g^{(2)}(\mu_0)+\delta\mu\, g_0^{(1)}(\mu_0)$$

↓

platí: $A = A_0 + \delta A$, $B = B_0 + \delta B$ $\quad \delta(AB) = AB - A_0B_0 = (A_0+\delta A)(B_0+\delta B) - A_0B_0 = \delta A B_0 + \delta B A_0 + \cancel{\delta A \delta B}$

$$= \mu g^{(1)}(\mu) - \mu_0 g_0^{(1)}(\mu_0) = (\mu - \mu_0) g_0^{(1)}(\mu_0) = g_0^{(1)}(\mu_0)\,\delta\mu$$

$$\boxed{\delta U = \delta\mu\, g_0^{(1)}(\mu_0) - \delta g^{(2)}(\mu_0) - \delta\mu\, g_0^{(1)}(\mu_0) = \boxed{-\delta g^{(2)}(\mu_0)}}$$

$$g^{(2)}(E) = \iint g(E)\,dE\,dE = \frac{1}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\hbar\omega_c\,\frac{4}{3}\sum_{j=0}^{j_{max}}\left(E - \hbar\omega_c(j+\tfrac{1}{2})\right)^{3/2} =$$

$$= S\hbar\omega_c\sum_{j=0}^{j_{max}}\left(E - \hbar\omega_c(j+\tfrac{1}{2})\right)^{3/2} \quad ; \quad S = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}$$

$\omega_c \to 0$ (není mg. pole): j_{max} musí jít do ∞, sousední sčítance mají velmi blízké hodnoty $\Rightarrow$ přejdeme k $\int$, horní mez integrace odpovídá situaci, kdy $\sqrt{\ }$ se stává nereálnou: $E = \hbar\omega_c(j_{max}+\frac{1}{2}) = x_{max}$

$$g^{(2)}(E) = S\hbar\omega_c\int_{j=0}^{\approx\infty}\left(E - \hbar\omega_c(j+\tfrac{1}{2})\right)^{3/2}dj = S\int_0^E (E-x)^{3/2}dx = -S\left[(E-x)^{5/2}\right]_0^E\cdot\frac{2}{5} =$$

$\hbar\omega_c(j+\frac{1}{2}) = x$

$\hbar\omega_c\,dj = dx$

$$= \boxed{\frac{2}{5}SE^{5/2} = g_0^{(2)}(E)}$$

$$g_0(E) = \frac{3}{2}SE^{1/2}$$

$g^{(2)}(E)$ pro slabé pole ($\hbar\omega_c$ je malé):

krok stranou: spočteme $R = \Delta\sum_{j=0}^{j_{max}} f(t_j)$, $t_j = \Delta(j+\frac{1}{2})$

$$\Delta\cdot f(t_j) = \int_{\Delta j}^{\Delta(j+1)} f(t_j)\,dt \approx \int_{\Delta j}^{\Delta(j+1)}\left[f(t) - f'(t_j)(t-t_j) - \frac{1}{2}f''(t_j)(t-t_j)^2\right]dt =$$

$$-\frac{f'(t_j)}{2}\left[(t-t_j)^2\right]_{\Delta j}^{\Delta(j+1)} = -\frac{f'(t_j)}{2}\left[\left(\Delta(j+1) - \Delta(j+\tfrac{1}{2})\right)^2 - \left(\Delta j - \Delta(j+\tfrac{1}{2})\right)^2\right]$$

$$= -\frac{f'(t_j)}{2}\left[\left(\frac{\Delta}{2}\right)^2 - \left(\frac{\Delta}{2}\right)^2\right] = 0$$

$$= \int_{\Delta j}^{\Delta(j+1)} f(t)\,dt - \frac{1}{6}f''(t_j)\left[(t-t_j)^3\right]_{\Delta j}^{\Delta(j+1)} = \int_{\Delta j}^{\Delta(j+1)} f(t)\,dt - \frac{1}{6}f''(t_j)\,2\left(\frac{\Delta}{2}\right)^3 \quad \ldots\ 1\text{ člen}$$

$$R = \int_0^{\Delta(j_{max}+1)} f(t)\,dt - \frac{1}{6}\left(\frac{\Delta}{2}\right)^2\underbrace{\Delta\sum_{j=0}^{j_{max}} f''(t_j)}_{\text{stejný postup } = \int_0^{\Delta(j_{max}+1)} f''(t)\,dt + (\sim \cancel{f''''})} = \int_0^T f(t)\,dt - \frac{1}{6}\left(\frac{\Delta}{2}\right)^2\left[f'(T) - f'(0)\right] \quad ; \quad T = \Delta(j_{max}+1)$$

proč jsme to dělali?:

$R = g^{(2)}(E)$, $\Delta = \hbar\omega_c$, $f(t) = S(E-t)^{3/2}$, $f'(t) = -\frac{3}{2}S(E-t)^{1/2}$, $T = E$

$$\boxed{g^{(2)}(E) = \int_0^E S(E-t)^{3/2}\,dt} - \frac{1}{6}\left(\frac{\hbar\omega_c}{2}\right)^2\left[\frac{3}{2}SE^{1/2}\right] = \boxed{g_0^{(2)}(E) - \frac{1}{4}\left(\frac{\hbar\omega_c}{2}\right)^2 SE^{1/2}}$$

$$\boxed{\delta U} = -\delta g^{(2)}(\underset{E_F}{\mu_0}) = -\left[g^{(2)}(E_F) - g_0^{(2)}(E_F)\right] = \frac{1}{4}\left(\frac{\hbar\omega_c}{2}\right)^2 SE_F^{1/2} = \frac{3}{2}SE_F^{1/2}\cdot\frac{1}{6}\mu_B^2B^2 = \boxed{\frac{1}{6}\mu_B^2B^2 g_F}$$

$$\hbar\omega_c = -\frac{eB\hbar}{m} = 2\mu_B B$$

$$-\chi\frac{B^2}{2\mu_0} = \frac{1}{6}\mu_B^2B^2 g_F \Rightarrow \boxed{\chi_{LANDAU} = -\frac{1}{3}\mu_0\mu_B^2 g_F = -\frac{1}{3}\chi_{PAULI}}\ T=0$$

Pozn.: Teplotní závislost se získá stejným fíglem jako pro μ a c_V